Eight Arguments Against the Law of Conservation of Momentum and Against Newton's Laws of Motion

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Argument #1. If the thermal radiation counts in the conservation of kinetic energy, it must also count in the conservation of momentum

It is universally accepted notion in physics today that the kinetic energy is conserved in all collisions by taking into consideration the generated thermal radiation. For example, when two bodies of masses m_1 and m_2 collide moving at velocities v_1 and v_2, their total *initial* kinetic energy is:

$$KE_i = \frac{m_1 v_{1i}^2}{2} + \frac{m_2 v_{2i}^2}{2} \quad (1)$$

That is, the total *initial* kinetic energy of the two bodies is equal to the total final kinetic energy of the two bodies moving with final velocities v_{1f} and v_{2f}, plus the energy of the newly generated thermal radiation, $m_R c^2$, that is,

$$KE_i = KE_f = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2} + m_R c^2 \quad (2)$$

However, it is also universally accepted notion that in the momentum conservation formula, momentum is conserved in the motion of the two interacting bodies alone, without taking thermal radiation into account. The total *initial* momentum of the two colliding bodies,

$$P_i = m_1 v_{1i} + m_2 v_{2i} \quad (3)$$

is assumed to be equal to the sum of the *final* momenta of the two colliding bodies alone:

$$P_i = P_f = m_1 v_{1f} + m_2 v_{2f} \quad (4)$$

Thermal radiation is totally neglected in this formula. It is assumed not to occur, or not count, when the total final momentum is calculated.

How is that possible?

Forces, momentum and kinetic energy are intimately connected and mutually dependent. All three have the same components, mass m and velocity v: in momentum $P=mv$, in kinetic energy $KE=mv^2/2$ and in force $F=ma$, where $a=dv/dt$.

Momentum carried by the thermal radiation in collisions is not always negligible. Even though the mass of the emitted thermal radiation maybe very small, it is multiplied by the speed of light, that is, 300 million times. The reason we are getting heat in our cars is because not all molecular motions and forces in the piston chamber can be converted into motion of the car. Some are absorbed by the engine block and emitted as heat.

According to the present laws of motion and the conservation laws, the new final velocities that are in the kinetic energy conservation equation *(Eq. 2)* allow room for the thermal radiation to be included *(Fig 1)*. However, the same velocities do not allow any room for the momentum of the thermal radiation to be included in the momentum conservation formula.

If we are to include momentum of the thermal radiation, then the total final momentum would be greater than the initial one. Furthermore, we would than have to change the velocities in the momentum conservation equation. If we do that, then the velocities in the kinetic energy conservation equation would also have to change. We would be then contradicting present conservation laws and the laws of motion from which the conservation laws are derived.

Why do we have this discrepancy?

Arguments that follow will explain.

Argument #2. Confusion about the role of internal forces in the law of conservation of momentum

A representative example of the alleged confusion about the role of internal forces in the final speeds of the colliding bodies is shown in the following quote by Hans Ohanian in his college textbook *Principles of Physics*. He wrote:

"... an external force is any force exerted on the body by some *other* body. By contrast, internal forces are those exerted by one part of the body on another part of the same body. *For instance, the forces that the screws or bolts in the sailboat exert on its planks are internal forces: such internal forces do not affect the motion of the boat.*" [1] (Emphasis added)

What is wrong with the above quote?

The forces between screws and planks in Ohanian example *existed before the sailboat is put into motion*, and before any forces acted on the boat. These types of internal forces certainly do not affect putting the boat into motion or changing its motion.

There is a different type of internal motions that *do not exist* prior to the action of a force. Will these *newly created forces* affect the final speeds of the bodies on which a force is impressed?

Let us consider a head-on collision of two balls of equal mass, $m_1=m_2=2\ kg$, where one ball is moving at a velocity $v_1=4\ m/s$, while the second ball is at rest $(v_2=0)$, as shown below.

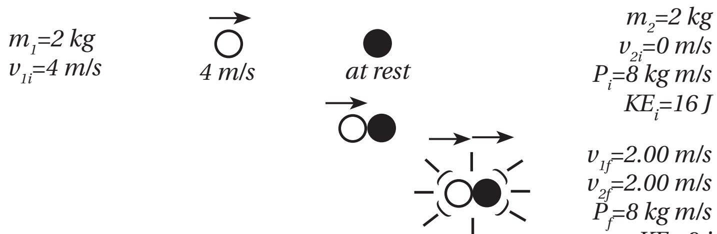

Figure 1

The two balls collide, compress against each other and, as a result, the two balls begin to vibrate and emit thermal radiation. After the collision, the two balls get joined together and both travel at *2 m/s*, as mandated by the law of conservation of momentum. The total final momentum of *8 kg m/s* would remain the same as the initial one.

In this collision, internal stresses, internal molecular friction along with the generation of heat, and air displacement due to heat, are *newly created forces*, that did not exist prior to the collision. This paper will show that because they are *newly created forces and motions*, they should reduce the rebounding force; that is, they should affect the speeds of the balls after a collision.

If they did not affect the speeds, we would end up with a greater amount of forces and motions after a collision than before.

If the two joined balls collide again, the motion of the two balls would again be conserved in the motion of the colliding balls and extra internal stress, extra motion, extra vibrations, extra heat and extra free force will be generated. We will be perpetually creating something out of nothing.

Based on the faulty analogy expressed in the above quote regarding screws and bolts on a sailboat, Ohanian, in the same manner as other physicists, arrived at the following conclusion:

"If the system is isolated so that there are no external forces, then the mutual interparticle forces acting between pair of particles merely transfer momentum from one particle of the pair to the other, just as in the case of two particles. Since all the internal forces necessarily arise from such forces between pair of particles, these internal forces cannot change the total momentum. Consequently, the total momentum of an isolated system obeys the conservation law, P = [constant]." [2]

Argument #3. Contradictions in Newton's two types of collisions

From Newton's description of collisions in *Corollary III*, we can deduce that Newton considered two types of collisions.

Type 1. Collisions where there is **equal deduction** from the motion of both colliding bodies after a collision and

Type 2. Collisions where whatever motion is ***added*** to one colliding body is ***subtracted*** from the motion of the other so that the sum remains the same.

Type 1. Symmetrical collisions with equal deduction from the motion of both bodies.

Newton started his analyses of forces and motions by performing numerous pendulum experiments. In *Corollary III*, Newton explained his 2nd and 3rd laws of motion by elaborating upon the two types of collisions. We will call the following as the Type 1 collisions or symmetrical collisions. Newton wrote:

"If the bodies meet with contrary motions, there will be an ***equal deduction from the motions of both;*** and therefore, the difference of the motions directed towards opposite parts will remain the same." [3] (Emphases added.)

We will demonstrate this type of collision with the following example where two balls of equal constitution and mass (2 kg) collide head-on with the same but opposite speeds (2 m/s), as shown in the figure below.

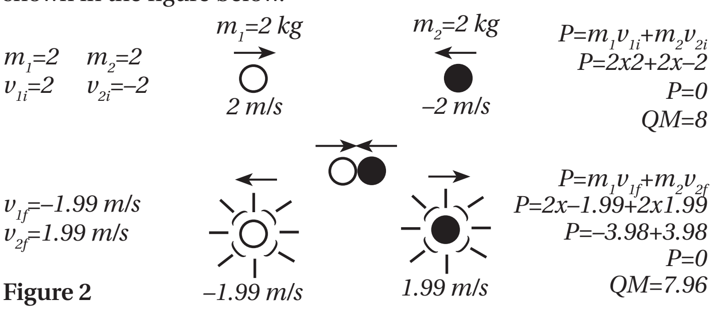

Figure 2

The two identical balls travel against each other, collide and rebound in opposite directions at slower speeds, 1.99 m/s.

Important fact here is that after the collision, as Newton tells us, there will be "an ***equal deduction*** from the motion of both."

This type of outcome of speed reduction cannot be challenged as it is universally accepted and experimentally proven.

Newton does not tell us why there are deductions and where they come from. Yet, equal deduction leads directly to non-conservation of the Quantity of Motion *(QM)*.

"Deduction from the motions of both" also means deduction in the sum of speeds, because the masses of the bodies remain the same. No external forces are present in this collision.

Newton's description of the above collision is in perfect agreement with Richard Feynman's description of the same symmetrical collision shown in *Fig. 2*. Feynman also gives us the account of why there is a deduction. He wrote:

"... two bodies of equal mass which collide with equal speeds and then rebound.... For a brief moment they are in contact and both are compressed... The bodies are immediately decompressed and fly apart again... the bodies fly apart with equal speeds." [4]

(Equality in speeds comes from symmetry in action.)

Feynman continued: "However, ***this speed of rebound is less***, in general, than the initial speed, because not all energy is available for explosion (rebound), depending on the material." [4]

"In the collision the rest of kinetic energy is transformed into heat and vibrational energy – the bodies are hot and vibrating."

Irrefutable facts in Newton's & Feynman's quotes:

1. A collision is a two-phase process consisting of a compression phase and a rebound phase.
2. A collision generates vibrations, stress waves and friction within the colliding bodies, which result in emission of heat.
3. Some of the initial *force, motion and energy* get dissipated during a collision through vibrations, friction and generated heat.
4. When Feynman states that "not all energy is available for explosion (rebound)," it can also be said that not all initial forces and motions are available for the collision's rebounding phase.
5. Hence, the sum of speeds of the two balls after a collision shown in *Fig.2* is smaller than before the collision.
6. According to Feynman, the amount of the speed loss and loss of motion depend on the ***material*** the balls are made of.
7. There are speed losses without any acting external forces.
8. According to Newton and Feynman, the new ***internal motions generated within the system do count*** in the formation of the final speeds of the two balls, contradicting the current belief that internal forces and motions have no effect on final speeds.

The above eight points should apply to all collisions in the universe and whenever a force is impressed on an object.

The current understanding of the concept of momentum does not tell the full story of what is happening in collision in *Fig. 2,* where momentum is conserved yet the quantity of motion is not. This is especially evident when the collision in *Fig. 2* is demonstrated with a ball in a box of the same mass (2 kg) and constitution colliding with equal but opposite speeds.

In *Fig. 3a*, the box moves to the right, while the ball moves to the left. They collide in *Fig. 3*b. The rebound speeds of the two balls are now slower, 1.99 m/s, and of opposite direction *(Fig. 3c)*. The box is now moving to the left and the ball to the right. They collide again *(Fig. 3d)* and rebound at even slower speeds, 1.98 m/s *(Fig. 3e)*.

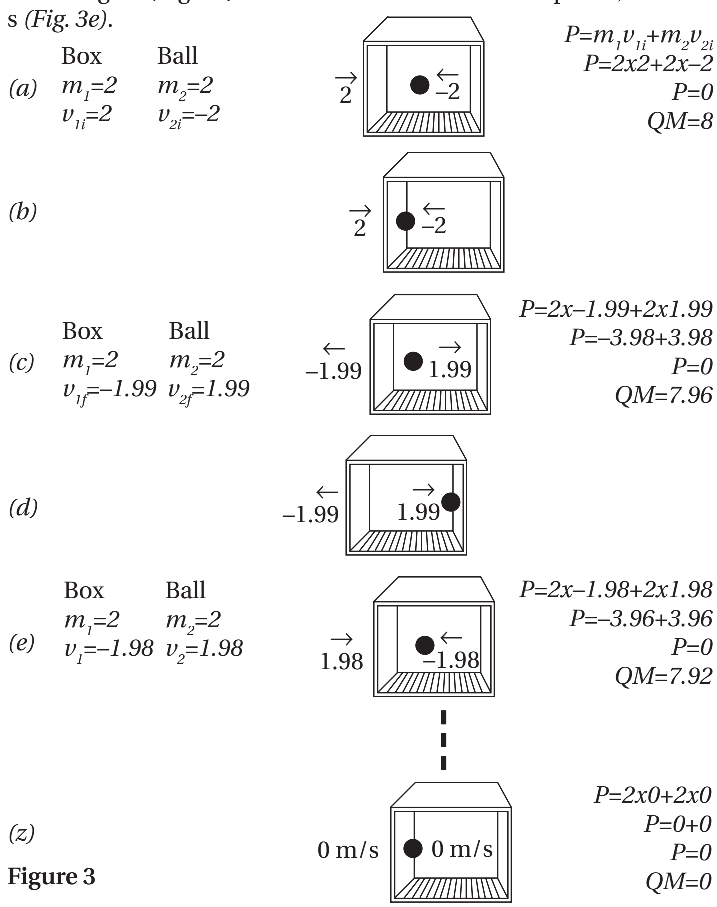

Figure 3

The ball and the box would continue colliding and loosing speed with every collision until they come to a stop, loosing all of their speed and the quantity of motion without action of any external forces, only the internal ones *(Fig. 3z)*.

Newton did not take into consideration this type of collisions when he formulated his laws of motion. He based his laws of motion on the second type, which is elaborated next.

Type 2. Collisions where whatever motion is added to one body is deducted from the motion of the other, so that the sum of motions remains the same.

In order to explain his laws of motion, Newton presented in his *Principia* six *Corollaries* and a *Scholium*. In *Corollary III,* he presented a representative example of a collision that is supposed to explain his 2nd and 3rd law of motion, where “whatever is ***added*** to the motion of the preceding body will be ***subtracted*** from the motions of the one that follows; so that the sum will be the same as before.” [17] He wrote:

"Thus, if a spherical body A is 3 times greater than the spherical body B, and has a velocity = 2, and B follows in the same direction with a velocity = 10, then motion of A : motion of B = 6 : 10.

“Suppose, then, their motion to be of 6 parts and of 10 parts, and the sum will be 16. Therefore, upon the meeting of the bodies ... if the body A acquires ... 12 parts of motion, and therefore after meeting proceed with ... 18 parts, the body B, losing so many parts as A has got, ... will go back with 2 (–2) parts ..." [3]

Let us present this collision with a box and a ball in it, as it was done when explaining Newton’s Type 1 collisions.

Fig. a: Body A is a ball with a mass of 3 kg (3 parts) moving to the right at a speed of 2 m/s (2 parts). Body B is a box of 1 kg (1 part) with the ball placed inside, also moving to the right at a speed of 10 m/s (10 parts). As is the case in Newton's example, motion of A : motion of B = 6 : 10. The initial sum of motion (momentum) of the two bodies is 16 kg m/s (16 parts).

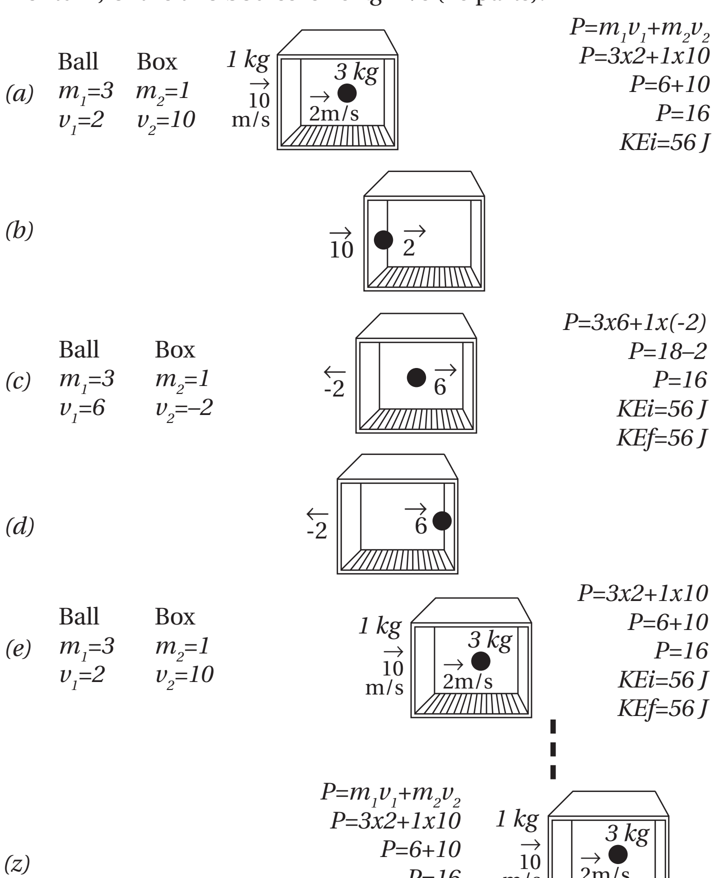

Figure 4

Fig. b: Because the box is moving at a faster speed, its left side will collide with the ball.

Fig. c: After the collision, the ball will gain 12 additional parts of motion (as indicated by Newton), yielding a total of 18 parts (6+12=18). However, the box will lose the same number of parts (12), and will end up with –2 parts of motion (10–12=–2). The sum of motions of the two bodies will be again 16 parts (18–2).

In other words, the sum of motion (momentum) of the two bodies after the first collision remains the same as the initial sum, 16 kg m/s.

The box will now travel at a slower speed but in the opposite direction, while the ball will gain speed after the first collision and will travel at a faster speed.

Fig. d: The ball and the box will collide once again.

Fig. e: After the second collision, the motion of the ball is reduced by 12 parts (18–12=6), but the motion of the box is increased by the same amount (–2+12=10). The sum of motions of the two bodies is again the same, 16 parts. The speed of the box is again 10 m/s and of the ball, 2 m/s, as it was before the first collision *(Fig.4a)*.

The speeds after the first collision in this example, 6 m/s and –2 m/s, can be obtained using kinematic equations for finding the final speeds when only initial speeds are known. [5] They are:

$$v_{1f} = v_{1i}[(m_1 - m_2)/(m_1+m_2)] + v_{2i}[(2m_2)/(m_1+m_2)]$$
$$v_{2f} = v_{1i}[(2m_1)/(m_1+m_2)] + v_{2i}[(m_2 - m_1)/(m_1+m_2)]$$

The above equations also yield the final speeds after the second collision, 2 and 10 m/s, which are the starting speeds in *Fig. 4a*. These speeds are to be expected, as the above equations are derived from elastic collisions that do no occur in nature. Hence, Newton’s representative collision also does not occur in nature.

According to Newton’s representative example, equal changes will continue indefinitely, and the sum of 16 parts of motion will always remain the same in all subsequent collisions. No motions and no forces are lost, and the box and the ball will continue colliding forever, *ad infinitum*.

Therefore, Newton’s example describes *perpetual motion*.

In Type 1 collisions, Newton allowed the reduction in speeds of both colliding bodies due to internal forces, resulting in the reduction of the total quantity of motion.

In Type 2 collisions, there is no reduction in the quantity of motion after a collision. Momentum is conserved in the motions of the colliding bodies alone, without taking into account any newly created forces, vibrations, internal stresses, internal frictions or emitted thermal radiation. There is no room for them in Newton’s Type 2 collisions.

From the outcomes in the Type 2 collisions, Newton came to very important observations, as expressed in *Corollary III:*

“The quantity of motion, which is obtained by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of the bodies among themselves,” and “... if the motions are directed toward the same parts, whatever is *added* to the motion of the preceding body will be *subtracted* from the motions of the one that follows; so that the sum will be the same as before.” Also, “...the sum of the conspiring motions ... and the difference of the contrary motions ... will always be equal to 16 parts, as they were before the meeting and reflection of the bodies.” [3]

Based on this understanding of the collision results, Newton formulated his 2nd and 3rd Laws of Motion: *“The change in motion is proportional to the motive force impressed ...”* and *“To every action there is always opposed an equal reaction.”* The word *proportional* refers to *no loss of motion* when the force of one body is impressed on another, as demonstrated by Newton’s representative example in *Fig. 4* and the conservation of 16 parts of motion.

This conservation leads to conclusion that Newton’s 2nd and 3rd laws describes perpetual motion that does not occur in nature.

If Newton’s laws describe motions that do not occur in nature, what are the laws that describe motions that *do* occur in nature?

Presently, there are no such laws!

Argument #4. Is it possible to conserve total quantity of motion after a few dozen collisions?

Let us consider collisions between a ball in a box similarly as in *Fig. 3*. The ball and the box are of the same mass, 2 kg. The ball is now initially at rest while the box is moving at 4 m/s.

Presently, the final speeds of the colliding bodies are found by taking into account the material the bodies are made of and the amount of deformation, by using the coefficient of restitution *e*, in conjunction with the law of conservation of momentum.

Because the ball and the box are made of the same material and travel at the same initial relative speed of 4 m/s, as in *Fig. 2*, we can determine the coefficient of restitution *e* from the resulting speeds in that figure. Coefficients of restitution *e* is the function of the final and initial speeds of separation and approach.

$$e=(v_{2f}-v_{1f})/(v_{1i}-v_{2i})$$

From collision in *Fig. 2*, e=0.995

Using the above equation and the law of conservation of momentum where

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f}$$

the final speeds after a collision are found. (See Calculations 1, p. 10) When the masses m_1 and m_2 are the same the speeds are:

$$v_{2f}=[v_{1i}(e+1)-v_{2i}(e-1)]/2 \qquad v_{1f}= v_{1i}+ v_{2i}- v_{2f}$$

These equations allow us to find the final speeds in collisions of the ball and the box, shown below.

In *Fig. 5a*, the box weighing 2 kg and moving at 4 m/s hits the ball at rest also weighing 2 kg. The collision causes a large amount of vibrations, deformation, internal friction and the generation of thermal radiation.

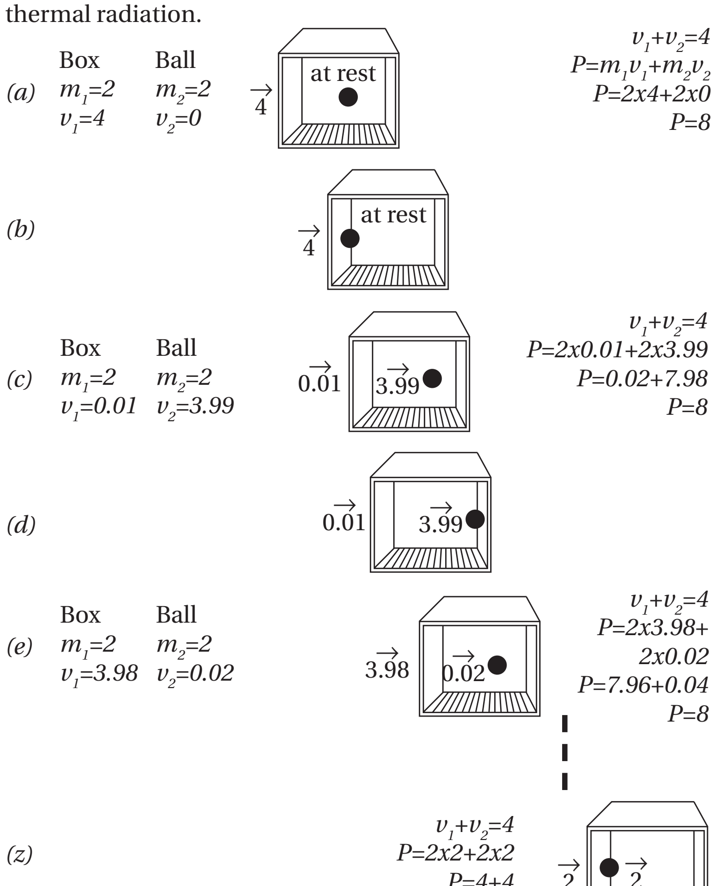

Figure 5

If these were an elastic collision (which do not occur in nature) the ball and the box would exchange the speeds after the collision. The box would come to rest and the ball would travel at 4 m/s in the same direction.

According to present theories found in every physics textbook, in order to maintain the conservation of momentum, whatever is added to the speed of the first body must be deducted from the motion of the other one, so that the total momentum of 8 kg m/s is maintained after each collision in *Figure 5*.

Hence, the speed of 0.01 m/s is added to the motion of the box *(Fig. 5c)* but subtracted from the motion of the ball, so that it travels at 3.99 m/s. The initial momentum of 8 kg m/s is thus conserved. The speeds are found using the coefficient of restitution *e* and the law of conservation of momentum. (See Calculations 1)

The box and the ball will be alternately increasing or decreasing their speeds after each collision.

At the end *(Fig. 5z)*, the box and the ball will stop colliding, as both would travel at a speed of 2 m/s.

Suppose the ball in the collision in *Fig. 5* is a wrecking ball, made of very strong metal weighing 2 tons, and the box is 2 meters on each side. They are made of the same metal and are of the same mass. The front and the back sides are open in order to be able to observe the collisions.

After numerous "Bang, Bang, Bang, Bang, Bang, Bang, ..., a few dozen times, and in spite of both bodies vibrating vigorously, in spite of deformation, in spite of generated thermal radiation, and in spite of air displacement around the warmer bodies, all created as new forces and new motions, the sum of the final momenta of the two colliding bodies would remain 8 tons m/s after each collision. The sum of speeds would also remains the same. No motion and no force would be lost whatsoever, according to current theories.

How is that possible?

The outcome here is the same as in Newton's numerical example where 16 parts of motion is maintained after each collision, except that the coefficient of restitution is added that is supposed to account for the speed changes due to the different material the ball and box are made of.

However, because the coefficient of restitution *e* is used in conjunction with the law of conservation of momentum, the coefficient of restitution is rendered ***unconsequential***, even though this concept acknowledges that the final speeds should be affected by the newly created internal motions and forces that depend on the type of *material* the bodies are made of.

As was the case with Newton's conservation of 16 parts of motion, shown in *Fig 4*, that does not leave any room for the newly created internal frictions, vibrations and thermal radiation to be included in the conservation of momentum, so is the case with the current understanding of the outcomes of collisions shown in *Fig. 5*.

We have to ask ourselves: *How is it possible to generate new additional forces and new additional motions from initial forces and motions yet conserve the initial ones?*

According to present theories, after a series of collisions in *Fig. 5*, there will be a greater amount of motion and force after each collision than before a collision occurs.

In this case, physicists started from established theories and laws to determine the outcomes, rather than to start from observations and then form or verify a theory.

Vibrations, stress waves, internal friction and thermal radiation carry forces. Yet, these forces are not accounted for anywhere. We are getting extra free force and free motion out of nothing.

This treatise challenges this assumption.

The stress waves are the sources of the rebounding forces. If the internal stress waves encounter resistance, they will lose some of their strength in overcoming it.

Present theories state that they do not count in the conservation of momentum. However, they never explain what is happening within the colliding bodies during a collision and how the rebounding force is formed.

That is the subject of the next argument.

Argument #5. Concept of dissipation of forces and motions and reduction of the rebounding forces

Toward the end of the previous argument a question was raised: *How is it possible to generate new additional motions and new additional forces from initial motions and forces yet conserve the initial ones?*

The answer was that this was impossible. Here is why:

Newton tell us that *"forces generate motions."* However, *motions generate forces*.

Every time a force of one body is impressed upon another body, like in a collision, there are two phases of action, as explained in Argument #3. There is a compression phase and the rebound phase. In the first phase, the compression occurs among the constituents of the two colliding bodies. It produces stress waves that travel through both bodies.

Werner Goldsmith wrote in his book *Impact The Theory of Physical Behaviors of Solid Bodies*:

"Forces created by collisions are exerted and removed in a very short interval of time and initiate stress waves which travel away from the point of contact." [6]

"... the disturbance generated at the contact point propagates into the interior of the bodies with a final velocity, and its reflection at bounding surfaces produces oscillations or vibrations in the solids." [7]

As the newly created stress waves travel through the bodies, the waves act as force on their constituents. They act on the nearest molecules which in turn exert force on the next ones in line. Friction is produced among the molecules. The waves travel through the bodies then rebound and travel in the opposite direction. They meet again forming the rebounding force.

When Feynman states that "... not all energy is available for explosion depending on the material", it is the same as saying that not all forces and motions are available for the rebounding phase. The internal structure of the material determines the outcome of a collision, meaning that internal forces play a role in the magnitude of the rebounding forces and, therefore, the final speeds.

It was argued in *Argument #1* that energy, momentum and forces are interdependent. Hence, if "not all energy is available for explosion" nor would forces nor motions.

The reduction of the rebounding forces would result in slowing rebounding speeds and loss of motion or momentum. This is the hallmark of every collision in the universe, and any time force is impressed on a body.

This reduction in the rebounding force is of enormous importance, as it will shape new understanding of the conservation laws and the laws of motion.

Argument #6. The 2nd law of thermodynamics proves that the law of conservation of momentum is untenable

The idea of dissipation of forces, motion and energy already exist in the 2nd law of thermodynamics. According to professor Hans Ohanian the law states:

"An engine operating in a cycle cannot transform heat into work without some other effect on its surroundings." [8]

In other words, the 2nd law of thermodynamics requires that dissipated energy in the surroundings must be taken into consideration when formulating this law.

The 2nd law of thermodynamics cannot be restricted to energy alone; it applies to forces and motion (momentum).

Contemporary physics separates energy from momentum and forces. It treats energy as a separate and independent physical phenomenon.

As stated earlier, forces, motions and energy are intimately connected. Forces generate motions, while motions are sources of forces and energy. If energy is conserved in a collision by taking the generated thermal radiation into consideration, this thermal radiation must be included in the conservation of momentum.

Although the laws of thermodynamics are related to thermal phenomena, they transcend forces and momentum, as thermal radiation carries both force and momentum. If energy is dissipated into surroundings, so will forces and momentum.

Argument #7. Difficulties in separating the loss of momentum due to external forces from those due to internal ones

The law of conservation of momentum dominated physics for a few centuries now. It was a given for a long time. Experiments to verify the law or to show non-conservation of momentum has not been seriously tried because it was universally assumed that the law of conservation of momentum is unassailable.

If some of the experiments showed discrepancies, these were immediately attributed to the effects of external forces. However, the effects of the external forces are difficult to measure, if not impossible. It appears that there were no accurate experiments performed to prove or verify the law of conservation of momentum and accurately measure the effects of external forces and discriminate them from the effects of internal ones.

Even the latest air tracks, where colliding gliders ride on a cushion of air, that are used in college and universities' class-rooms, are not totally frictionless. There is a turbulence of air that complicates the tests. The gliders pass through timing gates in order to measure their initial and final speeds and thus measure the initial and final momenta. The test results showed that final momentum was usually smaller than the initial one. However, the discrepancy was always attributed to the effects of external forces.

One manufacturer of air tracks performed experiments in their own lab and showed the results in the instruction manual. [9]

TABLE 1

m1 (g)	m2 (g)	v1i m/s	v2i m/s	v1f m/s	v2f m/s	Pi kg m/s	Pf kg m/s	Error %
180.2	201.3	0.46	0	–0.03	0.40	0.08	0.08	9.08
180.2	201.3	0.38	0	–0.03	0.34	0.07	0.06	9.44
180.2	201.3	0.52	0	–0.04	0.46	0.09	0.09	8.40
180.2	201.3	0.43	0	–0.04	0.38	0.08	0.07	10.43
180.2	201.3	0.53	0	–0.03	0.47	0.10	0.09	7.59

The glider with the mass of 180.2 g and initial speed of 0.46 m/s collided in the first try with another glider at rest with the mass of 201.3 g. The final speed of the first glider (v1f) was -0.03 m/s and the other was 0.40 m/s. There were 5 tests using the same gliders.

Here is the twist: The total initial momentum (Pi) and the total final momentum (Pf), shown in the shaded area in Table 1, were shown in *kilograms*, yielding no loss of momentum, or a small loss! Hence, Table 1 gives the impression that momentum is conserved.

However, when momentum is expressed in *grams* in Table 2, we see large differences between initial and final momenta.

TABLE 2

m1 (g)	m2 (g)	v1i m/s	v2i m/s	v1f m/s	v2f m/s	Pi g m/s	Pf g m/s	Error %
180.2	201.3	0.46	0	–0.03	0.40	82.89	75.11	9.38
180.2	201.3	0.38	0	–0.03	0.34	68.48	63.04	7.94
180.2	201.3	0.52	0	–0.04	0.46	93.70	85.39	8.87
180.2	201.3	0.43	0	–0.04	0.38	77.49	69.29	10.58
180.2	201.3	0.53	0	–0.03	0.47	95.51	89.21	6.60

The average margin of error of 9% is huge considering that the tests were performed on nearly frictionless air track, indicating that the error could not have been due only to external forces but to internal ones as well, causing the loss of rebounding forces and the possibility that momentum is not conserved in the motions of the colliding bodies alone.

Argument #8. Current equations for finding final speeds in a perfect collision contradict the law of conservation of momentum

Let us analyze a symmetrical collision and a perfect collision where the two balls get stuck together after a collision. The two collisions share some common characteristics: the balls are of the same weight and constitution and collide with the same relative speed of 4 m/s. The coefficient of restitution *e* is also the same. The first collision is the same as the one in *Argument 3, Fig. 2*, shown here again.

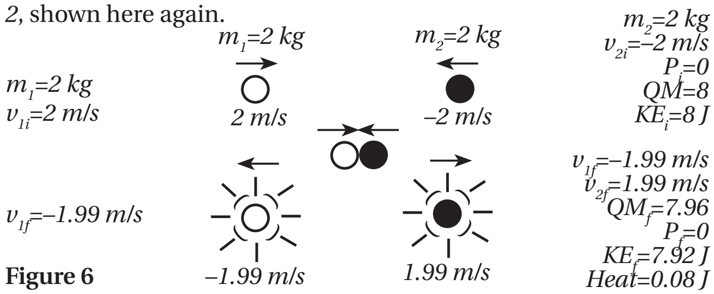

Figure 6

We showed in *Argument #4* that the coefficient *e* is the function of the final and initial speeds of separation and approach:

$$e=(v_{2f}-v_{1f})/(v_{1i}-v_{2i})$$

From the collision shown above, coefficient *e*=0.995.

The second collision is a perfect collision of the same two balls as in *Fig. 6*, except that one ball is at rest, while the other ball moves against it in a head-on collision with the same relative speed of 4 m/s *(Fig. 7)*. After the collision, the two balls are latched together and move to the right.

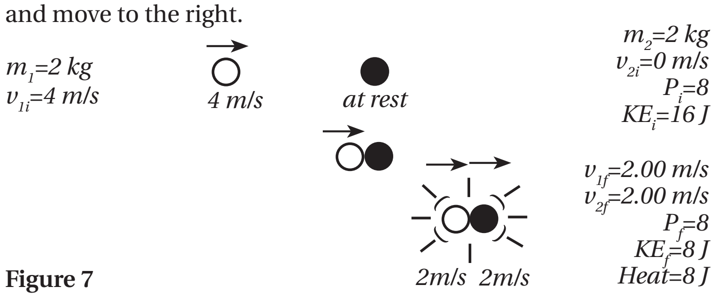

Figure 7

In order to conserve the initial momentum of 8 kg m/s, the two balls would have to move after the collision at 2 m/s, according to the law of conservation of momentum.

Let us find the final speed of the two joined balls using the equation for the coefficient of restitution *e* and the equation for the conservation of momentum. (See Calculations 2, p. 10) In *Argument #4* we showed that the equation for the final speeds are:

$$v_{2f}=[v_{1i}(e+1)-v_{2i}(e-1)]/2 \qquad v_{1f}= v_{1i}+ v_{2i}- v_{2f}$$

Because the initial speed of one ball was zero, the above equations for the speed of the two joined balls are reduced to:

$$2v_{1,2f}=v_{1i}(e+1)/2 \quad \text{or} \quad v_{1,2f}=[v_{1i}(e+1)/2]/2$$

$$v_{1,2f}=v_{1i}(e+1)/4$$

The initial speed of the first ball is 4 m/s and coefficient *e* is found to be 0.995. Hence,

$$v_{1,2f}=4(0.995+1)/4$$

$$v_{1,2f}=1.995 \text{ m/s}$$

Current equations for finding the final speeds in the above collision using the coefficient *e*, along with the law of conservation of momentum, yield the final speed to be less than 2 m/s, resulting in the smaller total final momentum (7.98 kg m/s) than the initial one (8 kg m/s). The loss of momentum is 0.02 kg m/s,

In other words, these equations show that momentum is not conserved in this collision.

Contradictions in finding the final speeds in perfect collisions using different materials.

Contradictions in how the final speeds are calculated in perfect collisions are especially evident in the following set of collisions shown below, where the two balls get joined together after a collision and travel to the right.

The same collision that was examined in the previous argument in Fig. #7 is examined here again but with three different outcomes.

A 2 kg ball moving at 4 m/s collides with another ball at rest of equal mass and constitution. After the collision the balls get stuck together. The three outcomes are shown in *Figures a, b* and *c*.

In *Fig. 8a*, the two balls just latch together after the collision without any deformation and travel at 2 m/s, as mandated by the law of conservation of momentum where the total initial momentum of 8 kg m/s must equal the total final momentum. The outcome is the same as in Fig 7 in the previous argument.

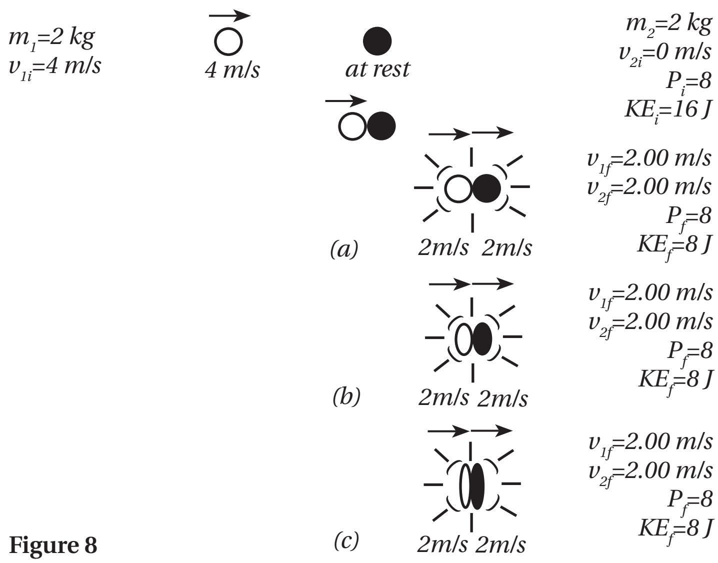

Figure 8

In *Fig. 8b*, the two balls are made of the softer material so that they suffer some deformation, get stuck together and travel after the collision at the same speed of 2 m/s, as mandated by the same law.

In *Fig. 8c*, the two balls are made of even softer material, soft clay, for example, so that they suffer greater deformation in the collision, get joint together after the collision and travel again at the same speed of 2 m/s, as mandated by the law of conservation of momentum.

Conservation of momentum of 8 kg m/s and the conservation of kinetic energy of 8 J after every collision in *Fig. 8*, regardless of the type of material the balls are made of, contradict Feynman's statement that in symmetrical collision in *Fig. 6*, the final speeds do depend of the material used.

Feynman wrote about the collision in *Fig. 6*: "However, ***this speed of rebound is less***, in general, than the initial speed, because not all energy is available for explosion (rebound), *depending on the material*." [4]

"In the collision the rest of kinetic energy is transformed into heat and vibrational energy—the bodies are hot and vibrating." [4]

However, in the set of collisions in *Fig. 8*, where balls get stuck together, neither heat nor vibrations could be produced, according to the law of conservation of momentum, because the final speeds are always the same, regardless of the material used, and because momentum is believed to be conserved in the motion of the two balls alone, leaving no room for the momentum carried by frictional, vibrational and thermal energy.

The coefficient *e* is supposed to account for the speed changes, but it did not. However, when we use different coefficient *e* for the collisions in *Fig. 8*, and the equations for finding final speeds in perfect collisions, the final speeds turn out to be slower than the initial ones.

Each of the three collisions in *Fig. 8* would have different coefficient *e* and, therefore, would have a different final speeds.

Equations for finding the final speeds in perfect collisions tells us that the greater deformation the smaller coefficient *e*, and the slower final speeds. Coefficient e=1 indicates the least amount of deformation and e=0 indicate the maximum.

For example, if the final speeds in collision in *Fig. 6* were –1.95 and 1.95 m/s, the coefficient *e* would be e=0.975. When this coefficient is applied to collision in *Fig. 8b*, where the same two balls collide with the same relative speed of 4 m/s, that is, with the same force, the final speed of the two joined balls would be 1.975 m/s, as calculated by using current equations for finding the final speed of the two colliding bodies.

There is a contradiction in the present methods for finding the final speed of the two joined balls in collision in *Fig. 8*.

When using the coefficient of restitution equation, the sum of the final speed of the two joined balls, 1.975+ 1.975 m/s, and the total final total momentum of 7.80 kg m/s are smaller than the initial ones, indicating that momentum is not conserved.

However, when using the law of conservation of momentum, as shown in *Fig. 8*, the final speeds remain the same, 2 m/s, as well as the total final momentum of 8 kg m/s, regardless of the materials the balls are made of.

The kinetic energy is also always the same, 8 J, indicating that these collisions cannot occur in nature, as the amount of the emitted thermal radiation would be the same in all three collisions.

Neither the total final kinetic energy of 8 J nor the total final momentum of 8 kg m/s could be the result in the three collisions in *Fig. 8*. Hence, the laws of conservation of momentum and energy would fail in these collisions.

If these laws fail to explain the outcomes of these collisions, what are the laws that could explain them?

It was stated in *Argument #3* that there are no such laws. That means we need to formulate new ones.

New Laws of Motion

Why would Newton elevate his definition of the measure of force to a law of motion?

Practically every physics textbook agree that *the essence of the second law is that force equals mass times acceleration, F=ma*. However, when we read Newton's own definition and explanation of his 2nd law in *Corollary III*, that follow the definitions of the laws of motion, we do not find anywhere the term *acceleration* or anything about the statement that *F=ma*. Newton wrote:

LAW II

"The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed." [10]

In *Corollary III* Newton also wrote:

"For action and its opposite reaction are equal, by Law III, and therefore, by ***Law II***, they produce in the motions equal changes towards opposite parts." [3]

"Therefore, if the motions are directed toward the same parts, whatever is *added* to the motion of the preceding body will be *subtracted* from the motions of the one that follows; so that the sum will be the same as before." [3]

"The quantity of motion, which is obtained by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of bodies among themselves." [3]

"If the bodies meet with contrary motions, there will be an **equal deduction from the motions of both**; and therefore, the difference of the motions directed towards opposite parts will remain the same." [3]

These important quotes from *Corollary III*, that describe the 2nd law, say nothing about equation *F=ma*. From Newton's description of the 2nd law, we can extrapolate equation *F=ma*, as was done by S. Chandrasekhar in his book *Newton's Principia for the Common Reader.*[6] [11] He wrote:

"In current terminology it (the 2nd law) states:

Force = change in **motion**
= change in (mass x **velocity**)
= mass x change in **velocity**
= mass x **acceleration"**

Indeed, *F=ma* can be extrapolated from the 2nd law, but this is not the essence of the law. Newton already defined the measure of force, *F=ma,* in *Definition VIII*, which precedes Newton's laws of motion. He defined the measure of momentum and the measure of force in the same sentence:

"For the quantity of motion (momentum) arises from the celerity multiplied by the quantity of matter *(P=mv)*; and the motive force arises from the accelerative force multiplied by the same quantity of matter, *(F=ma)*" [12] (Terms in parentheses added.)

Why would Newton define force, *F=ma*, as the 2nd law of motion when he already defined it in *Definitions*, and why would he elevate a definition of the measure of force to a law of motion?

What did Newton have in his mind when he formulated the 2nd law of motion?

The essence of Newton's 2nd law of motion

The above quotes explain Newton's 2nd law the best. They also show the richness of the law:

That forces **"***produce in the motion equal changes towards opposite parts***"** and that *"whatever is added to the motion of the preceding body will be subtracted from the motion of that which follows,"* along with the definition that *"The change of motion is proportional to the motive force impressed,"* is the essence of Newton's 2nd law.

Therefore, the 2nd law is about how the motions are ***generated*** *("any force generates motion," "triple the force triple the motion")* and how the generated motions are ***distributed*** *(added to or "subtracted from the motion of that which follows)," "the difference of the motions directed towards opposite parts will remain the same."*

The conservation of 16 parts of motion after each collision in Newton's numerical demonstration of the 2nd law shown in *Fig. 4* is the most accurate demonstration of Newton's law.

From these concepts of how motions are *generated* and how they are *distributed*, Newton formulated the 3rd law of motion—the law of equality of action and reaction. In other words, the 2nd law is the prelude to his 3rd law.

Newton's description of the 2nd law of motion that:

"... whatever is added to the motion of the preceding body will be subtracted from the motions of the one that follows; so that the sum will be the same as before,"

leads directly to the 3rd law of motion:

"To every action there is always opposed an equal reaction."

The two laws lead to the law of conservation of momentum:

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f}$$

It is easy to see that equating Newton's 2nd law of motion to the measure of force, *F=ma*, is obviously an error. There is nothing in equation *F=ma* that could lead either to Newton's 3rd law of motion or to the law of conservation of momentum. Mass *m* and acceleration *a* are just numbers.

It is also easy to see that the reduction of Newton's 2nd law to *F=ma* is not only improper and incorrect, it is an injustice to Newton and a great impoverishment of science.

Equation *F=ma* is simply a measure of force the way *P=mv* (mass times velocity) is the measure of momentum. It cannot be more than that and certainly not a law of motion.

Newton's errors – Prescription for perpetual motion

It was shown in *Argument #3* that there is a contradiction in Newton's two types of collisions. It was also shown that the numerical example, where the 16 parts of motion is conserved after each collision, leads to perpetual motion. In fact, Newton 2nd and 3rd laws of motion, along with the numerical example, that is, the law of conservation of momentum, is the best demonstration and definition of *perpetual motion*, where there is no loss of motion and no dissipation of forces when a force of one body is impressed upon another body. Hence, the balls in Newton's numerical example in *Fig. 4* would collide forever, add infinitum, according to Newton's 2nd and 3rd law of motion.

Perpetual motion is defined as:

"A state in which movement or action is or appears to be continuous and unceasing." [13]

"Perpetual motion is the motion of bodies that continues forever." [14]

"Perpetual motion, the action of a device that, once set in motion, would continue in motion forever, with no additional energy required to maintain it." [15]

Thus, the current mathematical expression of the law of conservation of momentum:

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f}$$

is also the mathematical expression of perpetual motion.

Furthermore, Newton thought that *equal changes toward contrary parts* would result in action to be equal to reaction.

Newton wrote: "... when the bodies concurred together directly, equal changes toward the contrary parts were produced in the motions and, ***of consequence***, *that the action and reaction were always equal."* [16] (Emphases added)

In the above quote, *"... equal changes toward the contrary parts ..."* refer to what happens after a collision, that is, it is all about reaction alone. There is no force of action in this quote, and, therefore, it is incorrect to conclude based on equal changes after a collision that "the action and reaction were always equal."

There can be equal changes in motions of the two colliding bodies relative to each other after a collision, but the sum of their final motions (reaction) can be smaller than the initial sum (action), otherwise we would have perpetual motion, as demonstrated by the conservation of 16 parts of motion in *Fig. 4*. In other words, "equal changes in motions" *do not lead to action being equal to reaction*.

Because perpetual motion does not exist in nature, Newton's 2nd and 3rd laws of motion that describe perpetual motion do not describe the real physical world of forces and motions.

New Laws of Motion that describe the real physical world

LAW II

The law of dissipation of forces, motions and energy

Whenever a body applies force upon another body, some of the initial force, energy and motions will be dissipated into newly created internal forces and motions, such as stress waves, internal friction, vibrations or in forces carried away by thermal radiation, which exerts force on the external environment, so that the rebounding force, or reactive force, will be smaller than the initial force, causing the total final quantity of motion of the two bodies to be smaller than the initial quantity.

Hence, because the initial force, energy and motion are dissipated so that some force is lost in the process of acting, and because *"the change of motion"* depends on the material the interacting bodies are made of, *"the change of motion"* cannot be *"proportional to the force impressed,"* as Newton stated in his 2nd law of motion.

LAW III

An (one) action can never be equal to a (one) reaction, or the mutual action of two bodies upon each other is never equal, because during action some forces, energy and motions are dissipated into vibrations and heat so that the rebounding force, or the force of reaction, is always smaller than the action force, as ***action is always equal to the sum of reactions****.*

New conservation laws

Law of conservation of momentum

The law of conservation of momentum states that momentum cannot be conserved in the motion of the colliding bodies alone unless momentum of internal motions, vibrations and thermal radiation are included.

As was the case with Newton's conservation of momentum that arises from his 2nd and 3rd laws of motion, the new law of conservation of momentum also arises from the two new laws of motion, which take into consideration the dissipation of forces and motions during a collision.

The current equation for the conservation of momentum

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f}$$

is replaced with the new one:

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f}+m_Rc$$

The total initial momentum is equal to the total final momentum plus the momentum carried away by the thermal radiation.

In this equation, the final speeds v_{1f} and v_{2f} are different than in the previous equation. The new speeds are the result of the dissipation of forces and motions during a collision that are expressed by the momentum of the generated thermal radiation, m_Rc.

Law of conservation of kinetic energy

The law of conservation of kinetic energy states that kinetic energy is conserved in the motion of the interacting bodies and in the energy of the emitted thermal radiation, with new final speeds that are the result of dissipated forces and motions into the environment.

Equation for the conservation of kinetic energy remains the same as the current one, except that the final speeds are different reflecting the new concept of dissipation of forces and motions during a collision.

$$KE_i = KE_f = m_1v_{1f}^2/2+m_2v_{2f}^2/2+m_Rc^2$$

The total initial kinetic energy is equal to the total final kinetic energy that includes the energy of the generated thermal radiation.

Law of non-conservation of angular momentum and correction in Newton's 1st law of motion

Present law

Here are a few representative definitions of the present understanding of the angular momentum and its conservation:

"... the angular momentum of a rotating body is defined as the product of its moment of inertia and its angular velocity. That is, the angular momentum *L*, with respect to a given axis, is defined as *L=Iw."* [17}

"... if the total external torque acting on a system is zero, then there is no change in the angular momentum of the system, and

the final angular momentum is equal to the initial angular momentum." [18}

"Because there is no torque acting on the earth, its angular momentum is conserved, and it will continue to ***spin with the same angular velocity forever."*** [18] (Emphasis added)

Indeed, will the earth "continue to spin with the same angular velocity ***forever?"***

The diminishing angular accelerative force

All physics textbooks state that spinning motions involve continuous acceleration. But in order to have continuous acceleration, a constant force is supposed to be acting in order to produce it.

For the earth to spin forever at the same velocity, there has to be a constant resupply of the accelerative force.

Bulging of the earth around the equator might be an argument against the current understanding of the law of conservation of angular momentum. This law led to the belief that the earth would spin forever with the same angular velocity, as quoted above.

Newton was the first to discover earth's bulging along the equator that is due to its circular motion around its own axis.

In order for constituents of the earth to migrate toward the equator, there must be a continuous force acting on them, pushing or pulling them toward the equator, while overcoming the resistance and friction of other constituents.

In the spinning of the planets, however, there is no such constant supply. The planets got their angular velocities from external forces billions of years ago during their formation. No such external forces are now acting on the planets.

There cannot be constant acceleration, that is, constant application of force without constant resupply of the force. If there is no resupply of the force, there will be constant reduction of the accelerative force and, therefore, the reduction of the angular velocity.

The same force that is used to displace the constituents of the earth toward the equator is also the force that is used to provide constant angular acceleration.

The use of existing force results in its reduction, and the reduction o f the available force would result in the slowing of the earth's rotational velocity.

This reduction of the angular force would be very small so that the change in the angular velocity would be also very small, and in many cases unobservable.

It might take thousands of years for a planet to show a measurable change in its rotational speed about its own axis. A million years ago the earth was probably spinning at a faster velocity.

Displacement of matter toward the equator involves friction and generation of heat. If the accelerative force is conserved, then, there will be a surplus of force carried by heat.

When the initial force is applied on a body so that the body acquires spinning motion, the initial force is transferred to the constituents of the spinning body. These constituents press, or apply force on the nearest constituents in the direction of the initial force.

How does the extra mass along the equator affect the angular velocity of a planet? Adding more mass along the equator must affect the rotational velocity of the earth because the overall distance of all the mass from the center of rotation has changed.

The outmost parts spin faster and posses much greater inertia than the other parts of the earth, or a top, for that matter.

These outermost parts may be pulling the other parts while spinning. But if the outmost parts are applying force on the other parts, there will be some dissipation of forces and motions, so that the available force that causes spinning might be gradually reduced.

These arguments allow us to propose a new law that covers angular motions.

New law of non-conservation of angular momentum

Angular momentum cannot be conserved because the angular accelerative force is continually applied on the constituents of the spinning bodies; however, because there is no continuous resupply of this force, the spinning bodies, a top or a planet, will be gradually loosing their angular velocity albeit over a longer period of time.

We might be able to test this new law by placing two tubes filled with water into an air-less freezer. One tube would be rotating at a high speed, while the other one is at rest. We would then observe at what temperature the water would freeze in the two tubes.

If the water freezes in the tube standing still first, this would indicate the generation of heat in the rotating tube and, thus, the dissipation of the initial angular force. It will also confirm that the new 2nd law of motion also applies to angular motions.

Newton's 1st law of motion and the law of non-conservation of angular momentum

The new law of non-conservation of the angular momentum would require an adjustment in Newton's 1st law of motion, or the law of inertia, which was first proposed by Galileo.

Right after defining the 1st law of motion, and in the next paragraph, Newton stated that: "A top, whose parts by their cohesion are continually drawn aside from rectilinear motion, ***does not cease its rotation***, otherwise than as it is retarded by the air. " [10] (Emphasis added)

In other words, Newton's law of inertia applies to angular motions as well. However, in the next sentence Newton wrote: "The greater bodies of planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular ***for a much longer time."*** [10]

Newton used the words: "for a much longer time," in the case of planets and comets. He did not use categorical words, instead he used vague ones: "for a much longer time."

What does this mean?

Having discovered the bulging of the earth along the equator, Newton might have anticipated the possible reduction of the angular velocities so he used the words "for a much longer time," when explaining his 1st law of motion.

This contrary meanings in terms *"does not cease its rotation,"* used in the spinning of the top, and *"much longer time,"* used in describing motions of the planets, pose a problem on how to understand the scope of Newton's first law. Newton defined the 1st law in one sentence:

"LAW I

"Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it." [10]

Arguments presented here against the law of conservation of angular momentum lead to an understanding that the new 1st law of motion is the restricted Newton's 1st law that applies only to bodies moving with uniform motion in the straight line, as defined in the above definition.

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[15] Britanica, Web
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[18] Peter Nolan, *Fundamentals of College Physics*, p. 260.

Calculations

1. Equations for finding the final speeds of the two colliding balls using the coefficient of restitution *e* in *Fig. 5*

Coefficient e is equal to the velocity of separation divided by the velocity of approach.

$$e=V_S/V_A \quad (1)$$

$$e=(v_{2f}-v_{1f}) / (v_{1i}-v_{2i}) \quad (2)$$

Equation for the conservation of momentum is:

$$m_1v_{1i}+m_2v_{2i} = m_1v_{1f}+m_2v_{2f} \quad (3)$$

When the masses m_1 and m_2 are the same, the above equation is reduced to:

$$v_{1i}+v_{2i} = v_{1f}+v_{2f} \quad (4)$$

$$v_{2f} = v_{1i}+v_{2i}-v_{1f} \quad (5)$$

$$v_{1f} = v_{1i}+v_{2i}-v_{2f} \quad (6)$$

When the final velocity v_{1f} in *Eq. 2* is replaced with v_{1f} in *Eq. 6*. we get:

$$e = [v_{2f}-(v_{1i}+v_{2i}-v_{2f})] / (v_{1i}-v_{2i}) \quad (7)$$

$$e = v_{2f}-v_{1i}-v_{2i}+v_{2f} / (v_{1i}-v_{2i})$$

$$e = 2v_{2f}-v_{1i}-v_{2i} / (v_{1i}-v_{2i})$$

$$e(v_{1i}-v_{2i}) = 2v_{2f}-v_{1i}-v_{2i}$$

or

$$ev_{1i}-ev_{2i} = 2v_{2f}-v_{1i}-v_{2i}$$

Now we can find the final speed of the second ball:

$$-2v_{2f} = -ev_{1i}+ev_{2i}-v_{1i}-v_{2i}$$

$$2v_{2f} = ev_{1i}-ev_{2i}+v_{1i}+v_{2i}$$

$$v_{2f} = (ev_{1i}+v_{1i}-ev_{2i}+v_{2i}) / 2$$

$$v_{2f} = [v_{1i}(e+1)-v_{2i}(e-1)] / 2 \quad (8)$$

The final velocity v_{2f} can be calculated by knowing the coefficient e and the initial velocity of the two balls. Once the v_{2f} is known, the final velocity of the other ball can be calculated from *Eq. 6:*

$$v_{1f} = v_{1i}+v_{2i}-v_{2f}$$

In the collision in *Argument #4*, *Fig. 5*, the masses of the two balls are the same, 2 kg, as in collision in *Fig. 2*. The first ball travels at a speed of 4 m/s, against the ball that is at rest. The coefficient of restitution was found from collision in *Fig. 2* to be e=0.995.

When we place these values in *Eq. 8*, we get:

$$v_{2f} = [v_{1i}(e+1)-v_{2i}(e-1)] / 2$$
$$v_{2f} = [4(0.995+1)+0(1-0.995)] / 2$$
$$v_{2f} = 4(0.995+1) / 2$$
$$v_{2f} = 3.99$$

From *Eq. 6*, the velocity of the other ball is

$$v_{1f} = 4+0-3.99$$
$$v_{1f} = 0.01$$

The sum of the final speeds, 4 m/s, is the same as the initial one, 4 m/s. Thus, the initial and final momenta would also remain the same.

2. Equation to find the final speeds of the two colliding balls in a perfect collision in *Argument #8* using coefficient *e*

We will start with the general equation for finding the speed of the 2nd ball, *Eq. 8*, when the masses of the two balls are the same

$$v_{2f} = [v_{1i}(e+1)-v_{2i}(e-1)] / 2$$

In a perfect collision, the two balls get joined together and travel with the same speed after the collision. Hence,

$$2v_{1,2f} = [v_{1i}(e+1)-v_{2i}(e-1)] / 2$$

In a collision when the initial speed of the 2nd ball is zero, the term $v_{2i}(1-e)$ becomes zero. Hence,

$$2v_{1,2f} = v_{1i}(e+1) / 2$$

$$v_{1,2f} = [v_{1i}(e+1)]/2 / 2$$

Thus, the speed of the two joined balls will be:

$$v_{1,2f} = v_{1i}(e+1)/4 \quad (9)$$

In the collision in *Argument 8, Fig. 7*, the initial speed of the 1st ball was 4 m/s, while the other ball was at rest. The coefficient of restitutions e was 0.995. The final speed of the two joined balls will be:

$$v_{1,2f} = 4(0.995+1)/4$$

$$v_{1,2f} = 1.995 \ m/s$$

The sum of final speeds is slightly less than the sum of the initial speeds. Hence, the total final momentum is smaller than the initial one, which means that momentum is not conserved.